ALAMEDA MATH CIRCLE

Graph Theory 2

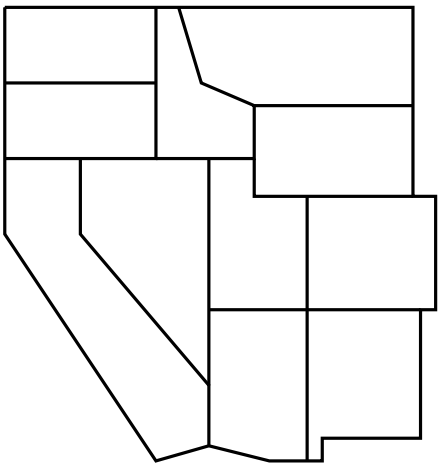
(Read the directions and follow the instructions)

A graph is a great tool for represent a **relationship** between **objects**.

Example: There are five sixth graders, Annie, Benjamin, Carmen, Dan, and Elizabeth. We know that Annie is friends with everyone, Benjamin is friends with Annie and Dan, and Carmen is friends with everyone except Benjamin.

Exercise 1: In the space below, draw a graph to represent the above situation. First, draw one vertex for Annie, and label it A. Then draw one vertex for Benjamin and label it B. Continue until you have one vertex for each person. Lastly, for every pair of people who are friends, draw an edge between their vertices.

Exercise 2: Below is a picture of the west coast of the United States. To the right of the map, draw a graph to represent this map, where each state gets a vertex, and you draw an edge between two vertices if those states have some border in common.



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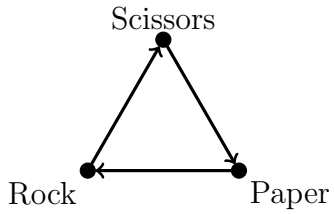
A **directed graph** is a graph where each edge is an *arrow* instead of a line. For example, the graph below represents the game Rock, Paper, Scissors:



Exercise 3: Draw a directed graph to represent the game 'Rock Paper Scissors Lizard Spock', described below. (Note: Lizard is formed by making your hand into a mouth, Spock is formed by spreading your fingers Star Trek style.)

Rock beats Paper and Lizard.

Paper beats Rock and Spock.

Scissors beats Paper and Lizard.

Lizard beats Spock and Paper.

Spock beats Rock and Scissors.

Exercise 4: In the space below, draw part of your family tree as a directed graph. An arrow should point from a parent to a child. Draw at least five people.

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Graph Theory 2

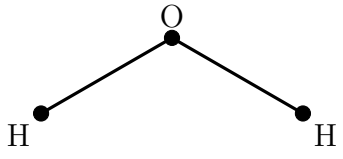
Graphs in Science

Phylogenetic Trees: Biologists who study evolution draw graphs to show how closely different species are related. Two species that are closely related have a **common ancestor**, a species that used to exist. At some point, that common ancestor evolved into two different species because groups in that species lived in different environments. This splitting is called **speciation**.

Exercise 5: Draw a directed graph to represent the situation described below. There should be a vertex for each species (including the common ancestors, which no longer exist), and an arrow connecting an older species to a species that it evolved into.

The **wolf** and **coyote** have a common ancestor, **species A**. The **badger** and the **otter** have common ancestor, **species B**. Species A and species B have a common ancestor, called **species C**. Finally, species C and the **panther** have common ancestor, **species D**.

Molecules: Chemists describe molecules by different kinds of atoms that make it up. For example, a molecule of **water** is made up of two atoms of **hydrogen** and one atom of **oxygen**. To describe this, chemists write H_2O. Atoms in a molecule are connected by **atomic bonds**. You can draw a graph to represent the molecule by drawing a vertex for every atom, and an edge for each atomic bond between atoms. There is a rule that hydrogen must have one atomic bond, while oxygen must have two. The graph is below:



Graph Theory 2

Exercise 6: Draw graphs that represent the following molecules:

NaCl (Salt.) Na stands for **sodium** and has one atomic bond. Cl stands for **chlorine** and has one atomic bond.

$\mathbf{CH_4}$ (Methane). C stands for **carbon** and has four atomic bonds.

$\mathbf{C_4H_{10}}$ (Butane).

$\mathbf{C_6H_6}$